Landon’s Questions about FFT & Signals

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1. An FFT takes a signal that is a function of time, *s(t)* and converts it into an equivalent function of frequency *s(f)*. This involves a transition from a possible real signal, into the complex plane - the resulting points in the FFT are complex numbers. What do the real/complex components of that number mean individually? What does the L2-norm of *that* complex number mean physically? What does the “angle” between the values mean?
2. Using the standard DFT matrix, this is an O(N2) operation. Thus an 8-pts DFT matrix has 64 elements in it. How does the decomposition of this matrix into the FFT work? How is the DFT *decomposed* to make an algorithm that is O(NlogN)? Whgat about this decomposition favors signals of lengths 2n? As per Scipy’s documentation pages on FFT, they actually only use the DFT! <https://docs.scipy.org/doc/scipy/reference/generated/scipy.fftpack.fft.html> Should look into learning how to write my own FFT?
3. In an 8-pt DFT, the 0-th term serves as the ‘0 oscillation term’, much like the a0 coefficient in the trigonometric series expansion. The 1-st and 7-th terms, and so forth are then complex conjugates of each other due to the symmetry of the unit circle? Have I completely misinterpreted this? Where does this rule break? Why can’t we just compute the first (N/2 + 1) and them compute the respective conjugates to save computation time?
4. In an FFT with sufficiently high frequency resolution and sufficiently distinct frequencies, what is the physical interpretation of the high of the isolated “spikes” in the spectrum. The spikes seem to be related to the amplitude of frequency component and the duration that it exists in the window. I see them referred to an “arbitrary amplitude units”. Is this power, energy, etc.? For example, if one where to integrate the *s(f)* function/array from *fa* to *fb*, what is the physical interpretation of the result?
5. Each column of the DFT matrix is a complex sinusoid oscillating with an integer number of periods in an N-pts sample window. What is the physical interpretation of each row? They are all also complex sinusoids with integer number of periods, where the exponent increments by the row number. This means that a discrete signal *s(t)* is projected into various frequencies? The idea behind the DFT is to test the “overlap” (inner product) of time-space and frequency-space? This this the correct interpretation of this operation?